Quiz 2 Solutions

1. Compute
$$\int_0^\pi (x+1)\sin(2x)\,\mathrm{d}x.$$

Solution: Integration by parts:
$$\begin{aligned}\int_0^\pi (x+1)\sin(2x)\,\mathrm{d}x &= (x+1)\frac{-\cos(2x)}{2}\Big|_0^\pi - \int_0^\pi 1\cdot\frac{-\cos(2x)}{2}\,\mathrm{d}x \\ &= (\pi+1)\frac{-1}{2} - (0+1)\frac{-1}{2} + \frac{\sin(2x)}{4}\Big|_0^\pi \\ &= -\frac{\pi}{2}\end{aligned}$$

2. Compute
$$\int \frac{1}{(x^2+1)^2}\,\mathrm{d}x.$$

Solution: Trig sub $x=\tan\theta$. Then
$$\begin{aligned}\int \frac{1}{(x^2+1)^2}\,\mathrm{d}x &= \int \frac{1}{(\tan^2\theta+1)^2}\sec^2\theta\,\mathrm{d}\theta \\ &= \int \frac{1}{\sec^2\theta}\,\mathrm{d}\theta \\ &= \int \cos^2\theta\,\mathrm{d}\theta \\ &= \int \frac{1}{2}+\frac{\cos(2\theta)}{2}\,\mathrm{d}\theta \\ &= \frac{1}{2}\theta + \frac{\sin(2\theta)}{4} + C \\ &= \frac{1}{2}\arctan x + \frac{1}{4}\sin(2\arctan x) + C \\ &= \frac{1}{2}\arctan x + \frac{x}{2(1+x^2)} + C\end{aligned}$$

(You don't have to simplify $\sin(2\arctan x)$ as in the last step)

3. Compute
$$\int \frac{2}{x^3-x}\,\mathrm{d}x.$$

Solution: Partial fractions: $x^3 - x = x(x-1)(x+1)$, so $\frac{2}{x^3-x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$. We can find $A = -2, B = 1, C = 1$ using the "cover-up" method. Then

$$\begin{aligned}\int \frac{2}{x^3-x}\,\mathrm{d}x &= \int -\frac{2}{x} + \frac{1}{x-1} + \frac{1}{x+1}\,\mathrm{d}x \\ &= -2\ln x + \ln(x-1) + \ln(x+1) + C\end{aligned}$$